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EQUATIONS FOR THE PHYSICAL PROCESS SIMULATION MODEL

COOPERATIVE STUDY ON DEVELOPMENT OF MODELS FOR PREDICTING SEDIMENT YIELD FROM SMALL WATERSHEDS

Prepared for

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by

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INTRODUCTION

The objectives of the study is to develop the best possible prediction models for estimating sediment yields from a broad spectrum of watersheds. The model is to include the routing of sediment from the supply areas and through the channel system.

The prediction models will estimate:

1. The amount of soil loss defined in terms of weight, volume, size and fractional mineral and organic matter from specified resource response units;
2. The amount of sediment transport out of the watersheds through principal drainage networks for winter and summer seasons, water years and major runoff events;
3. The effect of watershed treatment on sediment yields;
4. The amount of aggradation and degradation at different points in the channel system.

Our studies [1973] of other's work have led to the conclusion that the sediment model must be based on a few underlying physical principles in order to satisfy the objectives stated above. Herein, those equations describing the physical principles are presented. The universal soil loss equation is developed from the basic equations to illustrate the soundness of the type of approach to the sediment yield problem.

THE PHYSICAL PROCESS

In this study we intercept the hydrological cycle as the precipitation is about to reach the ground. An individual raindrop has grown to its full size and has attained its terminal velocity. As far as the sedimentation process is concerned, the most important consideration

is "where does the raindrop hit?" If the drop falls on a tree it is doubtful if that drop will contribute to the soil erosion process. In fact that drop is most likely to remain in the tree until it is evaporated and transported back into the atmosphere. When the drop does find a path through the tree to the ground most of kinetic energy has been lost in the tree and the drop has little erosional power. Those raindrops which do not encounter any vegetation as they fall to the ground are capable of dislodging sediments upon impact. Moreover, the great number of raindrops collect together to form a small torrent river which is capable of removing the dislodged sediments.

It is not practical to attempt to follow each raindrop as it enters the watershed. The physical process whereby raindrops cause soil erosion can be simulated by assuming aggregate behavior. That is, the surface of the watershed is divided into units with trees and units without trees. All raindrops that fall onto one or the other environment are assumed to have the same effect on the erosional process in that environment.

The impact of the raindrop on the surface of the ground is only the first step in the soil erosion process. Those raindrops which combine together to form the overland flow component of the hydrologic cycle develop more erosional power due to accelerations which occur on the sloping portions of watersheds. The equations which describe the physical process of overland and channel water and sediment flow are described below.

BASIC EQUATIONS

There are three basic differential equations for gradually varied unsteady flow over alluvial surfaces or in alluvial channels. The three equations are:

1. The differential equation of motion for sediment-laden water.

$$\frac{\partial \rho_m Q}{\partial t} + \frac{\partial \rho_m QV}{\partial x} + gA \cos\theta \frac{\partial \rho_m y}{\partial x}$$

$$= \rho_m gA \left(S_o - S_f + \frac{q_l U_l}{gA} \cos\phi\right)$$

2. The differential equation of continuity for water.

$$\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} - q_l = 0$$

3. The differential equation of continuity for sediment.

$$\frac{\partial Q_s}{\partial x} + \rho g \frac{\partial CA}{\partial t} + \rho_b gW \frac{\partial z}{\partial t} - q_s = 0$$

in which

t = time in sec,

x = downslope distance in ft,

Q = water flow rate in cfs,

V = average velocity of the flow in fps,

g = acceleration due to gravity in ft/sq sec,

A = cross-sectional area of the flow in sq ft,

y = the flow depth in ft,

S_o = the bed slope of the channel or overland flow unit in ft/ft,

S_f = the friction slope associated with the flow rate Q in ft/ft,

q = lateral inflow per unit of downslope distance in cfs/ft,

U_l = lateral inflow velocity in fps,

Q_s = sediment discharge rate in lb/sec,

C = the concentration of the sediment in the water in lbs of sediment per lb of water,

W = width of the bed section in ft,

z = bed elevation in ft,

q_s = lateral inflow of sediments in lb/ft sec,

ρ_m = density of the sediment-laden water in slugs/cu ft,

θ = angle between the x-direction and the horizontal direction,

ϕ = angle between the x-direction and the inflow velocity U_l,

ρ = density of the water in slug/cu ft,

ρ_b = bulb density of the sediments forming the bed in slugs/cu ft,

The assumptions made in deriving the equations are that: 1. the flow is one-dimensional; 2. the velocity is uniform over the cross section of flow; 3. hydrostatic pressure prevails at every point in the flow; and 4. the sediment-laden water is substantially homogeneous.

At this time it is impractical to consider overland and channel flow as two- or three-dimensional flow. It is beyond our resources to obtain the topography required to describe the boundaries for two- or three-dimensional flow and to obtain the computer storage necessary to solve the two- or three-dimensional flow equations for a watershed. The other three assumptions are not critical to obtaining satisfactory predictions with the three equations.

SUPPLEMENTARY EQUATIONS

The three basic differential equations describe the time varying motion of sediment-laden water provided we can reduce the number of unknown variables to three. That is, we must be able to describe all the variables in the three basic equations in terms of only three variables. Alternatively we can assign constant values to some of variables when the physical conditions warrant such assumptions.

Some of the supplementary equations which are employed to relate variables are as follows:

Friction Slope

The friction slope S_f is defined in terms of the Darcy-Weisbach friction factor as

$$S_f = \frac{f}{4R} \frac{V^2}{2g}$$

in which

f = the Darcy-Weisbach friction factor,

R = the hydraulic radius of the flow section in ft.

By definition

$$R = \frac{A}{P}$$

in which P is the wetted perimeter of the flow section in ft.

Friction Factor

The Darcy-Weisbach friction factor f for open channel or overland flow on rigid boundaries is a function of the roughness of the boundary, the depth of flow, and the flow Reynold's numbers N_R if there is no lateral inflow and if the boundary is essentially a plain surface. The flow Reynold's number is

$$N_R = \frac{Vy}{\nu}$$

in which ν is the kinematic viscosity of the fluid in sq ft/sec. The friction factor, Reynold's number, relative roughness relation is presented in any fluid mechanics textbook (for example, Vennard [1962] p. 287). For certain ranges of Reynold's number, the Darcy-Weisbach friction factor can be presented in equation form but no equation which can represent the relation for all Reynold's numbers has been developed.

Friction Factors with Rainfall

For shallow flows, the impact of a raindrop in the flow causes an energy loss in addition to that caused by the flow boundary. That impact loss has been determined experimentally by Shen and Li [1973]. Their equation for friction loss with rainfall is

$$f = \frac{k + 27.2\, i^{0.41}}{N_r}$$

if $N_r < 900$. Here k is a number representing the Darcy-Weisbach friction factor without rainfall and i is the rainfall intensity in in/hr.

For $N_r > 2000$ Shen and Li found that the friction factor was not affected by the rainfall.

Friction Factors for Other Conditions

Modifications of the Darcy-Weisbach friction factor are required for other conditions of flow. For example, Gessler [1970] has determined the friction factor for turbulent flow through porous media. His results could be employed for the cases in which we have overland flow through rocks and gravels. Schneider [1970] has determined through experiment, the friction factor for shallow flow through brush and trees. With modification, the same information is applicable for overland flow through ground cover. In alluvial channels with sand beds the friction factor must be modified to take into account the bed form loss caused when the bed is deformed into ripples and dunes. This adjustment in friction factor has been described by Stevens and Simons [1973].

Boundary Shear Stress

For open channel flow or overland flow the average boundary shear stress τ is defined as

$$\tau = \gamma R S_f$$

The terms R and S_f have been defined previously and γ is the unit weight of water in lbs/cu ft. Also

$$\gamma = \rho g$$

Shield's Parameter

The initiation of coarse sediment motion is described by Shield's parameter,

$$T = \frac{\tau_c}{g(\rho_s - \rho)d}$$

in which d is the particle grain size and τ_c is the critical shear stress at which particle motion begins.

The critical Shield's parameter is a function of the grain Reynold's number

$$N_R = \left(\frac{\tau}{\rho}\right)^{1/2} \frac{d}{\nu} \quad .$$

The critical Shield's parameter versus grain Reynold's number relation is presented by Graf [1971].

Bed-Load

For situations where the bed is composed of sand and gravel the Meyer-Peter and Mueller equation for bed-load transport is valid. The equation is

$$q_s = \frac{8(\tau - \tau_c)^{3/2}}{\rho^{1/2}}$$

in which q_s is the bed-load transport rate in lb/sec ft and $\tau-\tau_c$ is the excess shear over what is required to initiate motion.

Bed-Material Load

For channels transporting sand on the bed and in suspension the modified Einstein method of computing the bed-material load is employed. Chen [1973] has developed a technique that greatly reduces the time required to compute the Einstein bed load.

UNIVERSAL SOIL LOSS EQUATION

Li et al [1973] have shown that the universal soil loss equation which was developed by regression of experimental plot data can be obtained from the basic and supplementary equations. The universal soil loss equation can be written in the form

$$q_s \propto L^a S_o^b$$

in which q_s is the soil loss per unit length from a strip of fallow of length L and slope S_o. The exponents a and b were determined by experiments. Some values of a and b are listed below.

Investigator	Value of a	Value of b
Zingg [1940]	0.66	1.37
Musgrave [1947]	0.39	1.35
Wischmeier and Smith [1965]	0.50	--
Meyer and Monke [1965]	0.35 to 0.60	1.2 to 1.6
Meyer [1965]	1.0 to 1.5	1.50
Meyer and Kramer [1968]	0.50	1.40
Young and Mutchler [1969]	1.24	0.74
Kilino [1972]	1.04	1.66

For a constant rainfall rate i on a soil plot of length L and slope S_o the dynamic equation of motion for sediment-laden water can be reduced to

$$S_o = S_f$$

Recall that

$$S_f = \frac{fV^2}{8gy}$$

for a unit width of channel. We can write

$$S_o = \frac{fq^2}{8gy^3}$$

in which q is the flow rate per unit width of soil plot and y is the depth of flow. Then

$$y = \left(\frac{fq^2}{8gS_o}\right)^{1/3}$$

and

$$\tau = \gamma y S_o$$

$$= \gamma \left(\frac{fq^2}{8g}\right)^{1/3} S_o^{2/3}$$

The sediment pickup rate p_s can be represented in the form the sediment transport equation

$$p_s = \beta \tau^a \quad .$$

The rainfall rate is i and since there is no upstream or lateral inflow

$$i = \frac{dq}{dx}$$

or

$$q = ix$$

where x is the downslope distance from the head of the plot.

The average sediment yield E from a unit area of plot is

$$E = \frac{1}{L} \int_0^L p_s \, dx \quad .$$

After substitutions the sediment yield equation becomes

$$E = \frac{\beta}{L} \int_0^L \tau^a \, dx$$

$$= \beta \frac{\gamma^a S^{2a/3}}{L} \int_0^L \left(\frac{fq^2}{8g}\right)^{a/3} dx$$

Generally for fine-grained soil it has been found the $a = 2$ so

$$E = \frac{\beta \gamma^2 i^{4/3}}{(8g)^{1/3}} \frac{S_o^{4/3}}{L} \int_0^L f^{2/3} x^{4/3} \, dx$$

If the flow Reynold's number is less than 900,

$$f = \frac{k + 27.2 \, i^{0.41}}{q/\nu}$$

and

$$E = \frac{\beta \gamma^2 i^{2/3}}{(8g)^{1/3}} (k + 27.2 i^{0.41})^{2/3} \nu^{2/3} \frac{S_o^{4/3}}{L} \int_0^L x^{2/3} \, dx$$

or

$$E = C(\beta, i, k) \, L^{2/3} \, S_o^{4/3}$$

where

$$C(\beta, i, k) = \frac{3}{5} \frac{\gamma^2 \beta i^{2/3}}{(8g)^{1/3}} (k + 27.2 \, i^{0.41})^{2/3} \nu^{2/3}$$

Thus, for shallow flows on soil plots, the sediment yield per unit length is proportional to the two-thirds power of L and four-thirds power of S_o and to the coefficient C. That coefficient takes into account the erodibility of the soil β, the rainfall intensity i, and the Darcy-Weisbach friction factor represented by k.

These computed exponents for L and S compare well with most of those determined by regression of soil plot data.

Suppose that the flow depth on the plot is such that the flow Reynold's number is greater than 2000. Then, according to Shen and Li [1973]

$$f = \frac{0.26}{N_R^{0.25}}$$

if the soil surface behaves in a hydraulically smooth manner. The sediment yield equation becomes

$$E = \beta\gamma^2 \, i^{7/6} \left(\frac{0.26\nu}{8g}\right)^{0.25 \; 2/3} \left(\frac{S_o}{L}\right)^{4/3} \int_0^L x^{7/6} \, dx$$

or

$$E = C(\beta) \, i^{7/6} \, L^{7/6} \, S_o^{4/3}$$

where

$$C(\beta) = \frac{6}{13} \beta\gamma^2 \left(\frac{0.26\nu}{8g}\right)^{0.25 \; 2/3}$$

For deep flows on soil plots, the sediment yield per unit length is proportional to the seven-sixth power of L and to the four-thirds power of S_o and to the coefficient $C(\beta)$ which is a function only of the erodibility of the soil. The effect of the rainfall rate can be separated out. The sediment yield is proportional to the seven-sixth power of i.

In conclusion, the universal soil loss equation can be developed from basic physical laws and can be represented as

$$E = C(\beta,i,k) \, L^a \, S_o^{4/3} .$$

The exponent a varies between 2/3 and 7/6 depending on the depth of flow and the coefficient $C(\beta,i,k)$ is a function of the erodibility of the soil β, the rainfall intensity i, and the Darcy-Weisbach friction factor represented by k.

The exponents of L and S_o developed from the basic equations agree well with those determined from experimental data obtained from soil plots.

FLOOD ROUTING IN THE MISSISSIPPI RIVER

Chen [1973] has employed the three basic equations for water and sediment routing to compute flood hydrographs on the Mississippi River. An example of his success is shown in Fig. 1. The computed hydrograph was obtained by routing measured water and sediment from Vicksburg, Miss., a distance of 130 miles downstream to Red River Landing. The computed hydrograph was then compared with the measured hydrograph. In this reach of Lower Mississippi the computations indicate that there was very little deposition or erosion in the 130-mile river channel during the 1950 flood.

CONCLUSION

The basic equations describing water and sediment transport in alluvial channels and on alluvial planes have been employed to derive the universal soil loss equation. This derived equation compares well with those determined from experimental data. The same basic equations were employed successfully by Chen [1973] to route Mississippi River floods.

We conclude that the physical process simulation model which employs the basic equations of water and sediment motion is the most promising model to meet the objectives of this U.S. Rocky Mountain Forest and Range Experiment Station study.

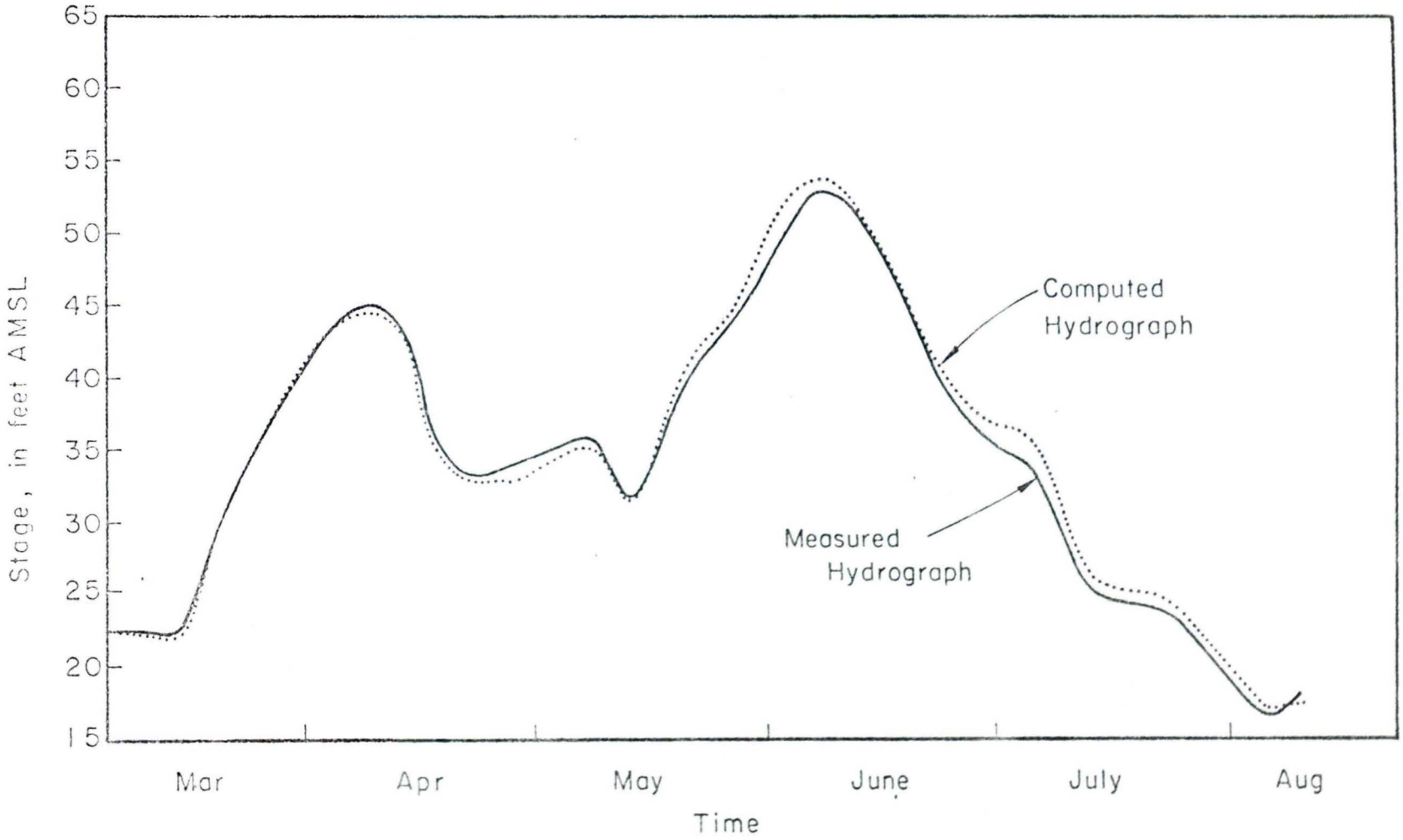

Fig. 1. Mississippi River, 1943 stage hydrographs at Red River Landing.

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